THE RUSSIAN PEASANT METHOD OF MULTIPLICATION.

By Joseph Bowden.

The Russian Peasant Method of Multiplication is said to be in common use in the villages of Russia, where it takes the place of the method which is in general use elsewhere.*

To be able to multiply by this method it is necessary to know how to add, how to double a number, and how to divide by two, obtaining the exact or lower approximate quotient.† Thus only a small part of the multiplication table need be known. The advantage of the method for persons who find this table difficult to remember is therefore obvious.

The method may be stated in the following rule:

The Russian Peasant Rule (Rule 1).—*Having given the positive integers a and b, to multiply a by b write down $a \times b$; under a write the exact or lower quotient obtained by dividing it by 2; under this quotient write the quotient obtained by dividing it by 2, and so on, until you obtain the quotient 1.*

Under b write its double, under this double its double, and so on, until you have as many numbers in the second column as in the first.

Next add the numbers in the second column which correspond to odd numbers in the first.

The result is the product of a and b.

The rule may be illustrated by the following examples:

* I am indebted for my knowledge of this method to Mr. L. Leland Locke, of the Brooklyn Training School for Teachers, and to Prof. David Eugene Smith, of Columbia University. Prof. Smith obtained the method, in a form slightly different from that in which I have presented it, in a clipping from a German newspaper, which quoted the method from the French journal *Cosmos*.

† See Bowden's "Elements of the Theory of Integers," § 513, The Macmillan Co., publishers, New York.

$45 \times$	24	$42 \times$	b—Omit
22	48—Omit	21	$2b$
11	96	10	$4b$—Omit
5	192	5	$8b$
2	384—Omit	2	$16b$—Omit
1	768	1	$32b$
	1080		$42b$

That this method is correct may be proved by means of the following theorem:

THE RADIX THEOREM.—*If a is any integer and t an integer numerically greater than one, if we divide a by t, obtaining the quotient q_0 and remainder r_0, then divide q_0 by t, obtaining the quotient q_1 and remainder r_1, then q_1 by t, obtaining the quotient q_2 and remainder r_2, and so on, after a time a number will be reached in the series a, q_0, q_1, q_2, $\cdots$, which is numerically less than t. Dividing it by t, the quotient zero may be taken.*

After the quotient zero is obtained all future quotients will be zero. If $a = 0$, the first quotient will be zero. If $a - = 0$, no quotient need be zero. But if the quotient zero is obtained (through necessity or choice, according as $a = 0$ or $a - = 0$), if we stop the process of division at this point, the series of remainders has the following properties:

If $a = 0$, there is only one remainder, which equals zero.

If $a - = 0$, an infinite number of sets of remainders can be found, the remainders are all numerically less than t, and the last remainder, r_n, is not zero.

Whether $a = 0$ or $a - = 0$, any set of remainders r_0, r_1, r_2, $\cdots$, r_n has the property that

$$a = r_0 + r_1 t + r_2 t^2 + \cdots + r_n t^n.$$

*If a is positive, $t > 1$, and when the division is inexact the lower quotient is taken, the quotients are all positive except the last, the remainders are all positive or zero, the last being positive, they are all less than t, and there is only one set of remainders.**

For example, if $a = 94$ and $t = 4$, we have the four quotients 23, 5, 1, 0 and corresponding remainders 2, 3, 1, 1.

* See Bowden, "Theory of Integers,' §§ 515, 516, 519, 526, 531, 607; Chrystal, "Text-Book of Algebra," Part I., p. 167, Adam and Charles Black, publishers, London.

Hence $94 = 2 + 3 \times 4 + 1 \times 4^2 + 1 \times 4^3$.

If $a=42$ and $t=2$, we have the six quotients 21, 10, 5, 2, 1, 0 and remainders 0, 1, 0, 1, 0, 1.

Hence $42=0+1\times 2+0\times 2^2+1\times 2^3+0\times 2^4+1\times 2^5$
$=2+2^3+2^5$.

Therefore $42b=(2+2^3+2^5)b=2b+8b+32b$.

This example proves the Russian Peasant Rule for any value of b when a is 42.

To prove the rule for all values of a and b suppose that a and b are positive and that we wish to multiply a by b. Let t be 2 and when the division is inexact choose always the lower quotient. Then we have the three series of numbers given in the three columns below:

$$\begin{array}{ccc} a & r_0 & b \\ q_0 & r_1 & 2b \\ q_1 & r_2 & 2^2b \\ q_2 & r_3 & 2^3b \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ q_{n-1} & r_n & 2^nb, \end{array}$$

the last column containing the number b and the numbers obtained from it by successive doubling.

Since the numbers $r_0, r_1, r_2, \cdots, r_n$ are all positive or zero and less than t, and t is 2, these numbers are all either 0 or 1. Moreover, if any number in the first column is even, the division is exact and the corresponding remainder, which is the corresponding number in the second column, is 0. If any number in the first column is odd, the division is inexact and the corresponding remainder, that is, the corresponding number in the second column, is 1. Moreover, since the last remainder, r_n, is not zero, it is equal to 1.

Now we have, by the Radix Theorem,

$$a=r_0+r_12+r_22^2+r_32^3+\cdots+r_n2^n.$$

Hence

$$ab=r_0b+r_1(2b)+r_2(2^2b)+r_3(2^3b)+\cdots+r_n(2^nb).$$

The r's are either 0 or 1. If any r is 0, the corresponding term can be crossed out. If an r is 1, it may be omitted as a factor from the corresponding term.

Thus ab is the sum of those of the terms in our last column,

$$b, 2b, 2^2b, 2^3b, \cdots, 2^nb,$$

which correspond to the value $r = 1$ in the second column, or to odd numbers in our first column.

Our rule is therefore proved to give the result asserted of it.

COROLLARY TO THE RADIX THEOREM.—*Every positive integer can be written as a sum of powers of 2 (including the zeroth power of 2, which is 1).*

E.g.,

$$14 = 2 + 2^2 + 2^3$$

$$47 = 1 + 2 + 2^2 + 2^3 + 2^5.$$

Some of my readers will probably be interested in the following two rules, which are variations upon the Russian Peasant Rule:

RULE 2.—*To multiply a by b write down in a vertical column the number a and the successive quotients, exact, lower, or upper, obtained by dividing it and the series of quotients by 2, until you obtain the quotient 0.*

In another vertical column, starting on a line with the first quotient, write down the number b and the numbers obtained from it by successive doubling, until you have a number in the second column for each quotient in the first. Next add the numbers in the second column which correspond to lower quotients and those which correspond to upper quotients. Subtract the latter sum from the former. The result is $a \times b$.

E.g.,	27			60		15
Upper	14	15		120		240
Exact	7	30		480		255
Lower	3	60		660		
Lower	1	120		255		
Upper	1	240	$27 \times 15 =$	405	Ans.	
Lower	0	480				

RULE 3.—*The same as Rule 2 with the following changes:*

Divide by 3 instead of by 2 and always choose the quotient which gives the remainder 0, 1, or — 1. Triple instead of double.

E.g.,	48		63	189
Exact	16	21	1701	567
Lower	5	63	1764	756
Upper	2	189	756	
Upper	1	567	$48 \times 21 = 1008$ Ans.	
Lower	0	1701		

Innumerable other rules may be derived from the radix theorem by giving t various values, positive and negative.

These other rules, however, are not so simple in statement as the three given, except the rule obtained by taking t equal to 10. This rule is practically the same as the ordinary rule for multiplication.

Examples.

1. Express as sums of powers of 2 the numbers 1, 2, 3, $\cdots$, 16.
2. Express as above the numbers 87, 96, 436, 783.
3. Multiply 96 by 87. Check by multiplying 87 by 96.
4. Multiply 783 by 436. Check by congruences to the modulus 5. Check also by modulus 6 and by modulus 4. (*If* $a \equiv b$ *and* $c \equiv d$, *then* $ac \equiv bd$.*)
5. Multiply 3,689 by 2,728. Check by casting out the nines.†
6. Multiply 359 by 422. Check by casting out the threes.
7. Multiply 8,979 by 7,639 and check by casting out the elevens.

$$[a \equiv (r_0 + r_2 + r_4 + \cdots) - (r_1 + r_3 + r_5 + \cdots).\ddagger]$$

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* See Bowden, "Theory of Integers," §§ 731, 753, 760; Chrystal, "Text-Book of Algebra," Part II., p. 500.

† See Chrystal, Part I., p. 175.

‡ See Chrystal, Part I., p. 178, ex. 27.